

Effects of Chemical Reaction on Stagnation Point MHD Flow over a Vertical Plane with Convective Boundary Conditions In The Presence of a Transverse Uniform Magnetic Field

¹ Adeniyani, A., ² Adigun, J. A

¹Department Of Mathematics, University Of Lagos, Lagos.

²Department Of Physical Sciences, Bells University Of Technology, Ota.

Abstract

A numerical investigation has been conducted on the convective plane stagnation point flow with convective boundary conditions. The mathematical model is presented for a two-dimensional steady, incompressible, electrically conducting and laminar free convection boundary layer flow over a vertical plate in a chemically reacting medium with an applied transverse magnetic field to the direction of flow. The basic partial differential equations are transformed and reduced to a system of non-linear ordinary differential equations by the use of the similarity transformation. The problem is addressed numerically, using fourth-order Runge-Kutta iterative scheme together with a shooting technique. It is observed that the velocity, temperature and concentration profiles are appreciably influenced by the presence of the transverse magnetic fields. We also discussed, graphically, the effects of the various hydromagnetic, heat and mass transfer parameters involved in the governing equations on the Skin-Friction coefficient, Nusselt and Sherwood numbers.

Keywords: convective boundary conditions, magnetic field, hydromagnetics, chemical reaction

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I. INTRODUCTION

Scientists and investigators have researched, so deeply, the flow near a stagnation point due to its technological and industrial applications. More recently, a renewed interest has been shown by some authors because of the much needed practical applications in metallurgy, including wire drawing, metal and polymer extrusion processes, particulate deposition on surfaces, cooling of nuclear reactors, magnetohydrodynamic power generation systems etc. Series of investigations have been done over the years by many researchers. Nield and Bejan [1], Ingham and Pop [2], Bejan and Khair [3], Sakiadis [4] and a few others have all researched into the subject matter. The Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in porous medium were looked into by Anghel et al. [5]. Crane [6] studied the two dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate.

Okedayo et al.[7] presented the effects of viscous dissipation in the mixed convection heat transfer over a plate with internal heat generation and convective boundary conditions and obtained local similarity variables. Adeniyani and Adigun [8] took studies on the effects of plane stagnation point flow with convective boundary conditions in the presence of a uniform magnetic field, using similarity solutions ignoring mass diffusion effects. Ibrahim Makinde [9] examined chemically reacting MHD boundary layer flow of heat and mass transfer past a low-heat-resistant sheet, moving vertically downwards in a viscous electrically conducting fluid permeated by uniform transverse magnetic field. Aman and Ishak.[10] carried out their analysis on stagnation flow towards a stretching plate in an electrically non-conducting fluid with convective boundary conditions. They considered ambient velocity, parallel to the plate to vary as the square root of distance along the plate. Cortell [11] examined nonlinear stretching. Omowaye and Koriko [12] reported the similarity solutions for free convection between parallel porous walls.

This study examines the effects of chemical reaction on stagnation point MHD flow over a vertical plane with convective boundary conditions. In the open literature within our reach, no author has considered this same type of problem. The effects of various thermo-magneto physical parameters on the velocity, temperature and concentration profiles are considered and discussed through tables and graphs.

II. PROBLEM FORMULATION

We consider two-dimensional stagnation point flow of a chemically reacting and electrically conducting incompressible viscous MHD stream in which a vertical plate is immersed. A uniform transverse magnetic field of magnitude B_0 is applied. It is assumed that this plate is impermeable, and the magnetic Reynold's number is negligibly small so that the induced magnetic field and Hall current effects are negligible. The wall temperature and the species concentration are designated by T_w and C_w respectively, while T_∞ and C_∞ are the ambient temperature and species concentration. The flow and heat transfer equations relevant for the model, namely the continuity, momentum, energy and chemical speed concentration equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} \beta_0^2 u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K_r (C - C_\infty) \tag{5}$$

Outside the boundary layer, use is made of the inviscid flow assumptions to eliminate the first term on the right of eqn.(2) to obtain a new form posited as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_\infty \frac{du_\infty}{dx} + \gamma \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} \beta_0^2 (u - u_\infty) \tag{6}$$

2.1 Boundary Conditions:

$$u(x, 0) = v(x, 0) = 0, -k \frac{\partial T}{\partial y}(x, 0) = h_f (T_f - T(x, 0)), C(x, 0) = C_w \tag{7a}$$

$$u(x, \infty) = ax = u_\infty, T(x, \infty) = T_\infty, C(x, \infty) = C_\infty$$

(7b)

In eqns. (1) – (7b) above, the velocity field $\mathbf{q} = (u(x, y), v(x, y))$ where u and v are the components of the velocity along and normal to the plate respectively. $T(x, y)$ is the temperature field, γ is the kinematic viscosity, α is the thermal diffusivity, h_f is the heat transfer coefficient, p is the fluid pressure, ρ is the fluid density, k is the thermal conductivity, σ is the electrical conductivity of the fluid, K_r is the reaction rate constant of the first order homogeneous and irreversible reaction, $\mathbf{B} = (0, B_0)$, is the imposed magnetic field, D_m is the mass diffusivity and T_f is the film temperature, conveniently taken as the mean temperature of the fluid.

In a bid to transform the above equations, we introduce the following similarity transformation/ parameters as follows:

2.2 Similarity parameters/ dimensionless numbers

$$\left. \begin{aligned} \eta &= y \sqrt{\frac{\alpha}{\gamma}}, & \psi(x, y) &= x \sqrt{\alpha \gamma} f(\eta), \\ Pr &= \frac{\gamma}{\alpha}, & Bi &= \frac{h_f}{k} \sqrt{\frac{\gamma}{\alpha}} \\ \phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty}, & \theta(\eta) &= \frac{T - T_\infty}{T_f - T_\infty}, \\ M &= \frac{\sigma B_0^2}{\rho \alpha}, & K &= \frac{K_r}{\alpha}, & Sc &= \frac{\gamma}{D_m} \end{aligned} \right\} \tag{8}$$

The velocity components of the flow are represented in terms of the stream function,

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

so that eqn. (1) is identically satisfied.

With the use of the similarity transformation, momentum, energy and chemical speed concentration equations become

$$f''' + ff'' - (f')^2 + 1 - M(f' - 1) = 0 \tag{10}$$

$$\theta'' + Pr f \theta' = 0 \tag{11}$$

$$\phi'' + Sc f \phi' - Sc K \phi = 0 \tag{12}$$

subject to the transformed boundary conditions:

$$f'(0) = f(0) = 0, -\theta'(0) = Bi(1 - \theta(0)), \phi(0) = 1 \tag{13a}$$

$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0. \tag{13b)}$$

Where the prime denotes differentiation with respect to η , Pr is the Prandtl number, Sc is the Schmidt number and M is the magnetic parameter.

The system of eqns. (10) – (13) is a coupled system of non-linear ordinary differential equations and it is difficult to solve by known available analytical methods.

III. NUMERICAL DISCUSSION AND RESULTS

The resulting equations governing the flow, heat and mass transfer together with the transformed boundary conditions have been solved numerically for various specified governing parameters Pr , M , Bi , Sc , K ; using the classical fourth order Runge-Kutta integration method alongside with a shooting technique carried out on a computer program which uses the symbolic and computational computer language in MAPLE(15). Figure 1 shows the numerical results for fixed $Pr = 0.72$, $Sc = 0.24$, $M = 0.1$ and $K = 0.1$ for a range of values of the Biot number $Bi = 0.1, 0.5, 1.0$ and 10.0 . It is observed that an increase in Biot number thickens the thermal boundary layer thickness and also increases the wall temperature. This results are in good agreement with Adeniyani and Adigun[8]. Also, when the magnetic parameter $M=0$, we recover exactly the results of Okedayo et al.[7]. The case for which the concentration species are not chemically reactive, i.e. $K=0$, agree well with results in the literature.

Figure 2 shows the effects of Prandtl number $Pr = 0.72, 2.71, 5.00$ and 7.10 for fixed $Sc = 0.24$, $M = 0.1$, $K = 0.1$ and $Br = 0.1$. We observe that as the Prandtl number increases, the thermal boundary layer thickness decreases together with the surface temperature. Figure 3 shows the effect of the magnetic parameter when varied, for values of $M = 0.1, 0.5, 1.0$ and 2.0 and fixed value of $Sc = 0.24$, $Bi = 0.1$, $K = 0.2$ and $Pr = 0.72$. It is observed that there is a decrease in the velocity boundary layer thickness and a fall in the velocity. The velocity fall is rapid near the wall after which it becomes gradual. From Figures 4 and 5, we see that the species concentration in the fluid had a maximum value at the plate surface and decreased exponentially to the free stream zero value away from the plate. The concentration boundary layer was observed to decrease with increasing Schmidt number in figure 4. Similarly in figure 5, an increase in reaction rate also reduces the concentration boundary layer.

In table 1, we present the computed values of Nusselt number, Sherwood number (representing heat and mass transfer rates at the surface respectively) and the Skin Friction Coefficient to see how they are being influenced by varying values of Magnetic parameter, Biot number, Schmidt number, reaction rate and Prandtl number. It is observed that an increase in the Magnetic parameter cause a reduction in the Skin Friction Coefficient at the surface of the plate. The prandtl number as being increased also reduces the Nusselt number but the reverse is observed when Biot number is increased. The Biot number is seen to increase the Nusselt number. Furthermore, we also observe that an increase in Schmidt number slightly increases the Sherwood number. Similarly the same is observed for varying the Reaction rate. An increase in the Reaction rate causes a corresponding increase in the Sherwood number.

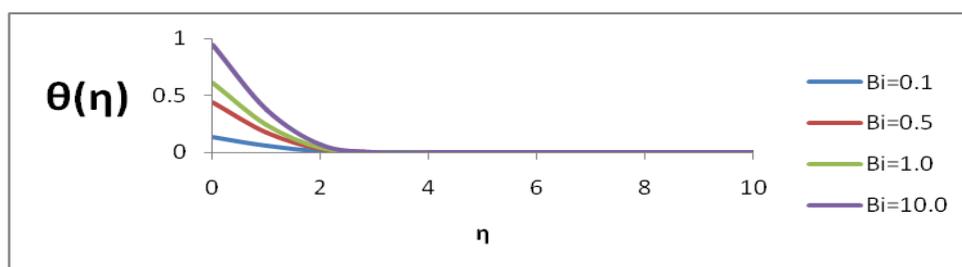


Figure 1: Effect of Biot number on the temperature profile for $Pr = 0.72$, $Sc = 0.24$, $M = 0.1$ and $K = 0.2$

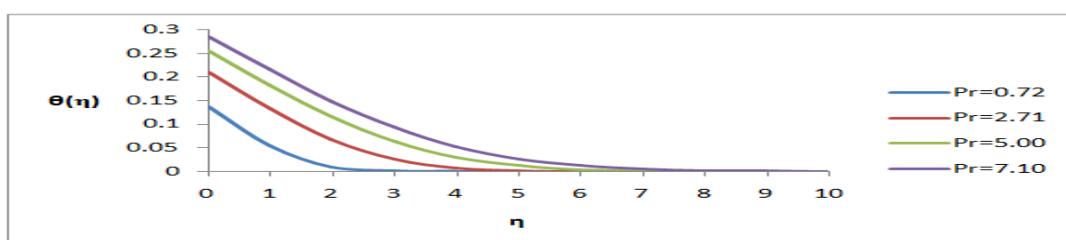


Figure 2: Effects of Prandtl number on the temperature profile for Bi = 0.1, Sc = 0.24, M = 0.1 and K = 0.2

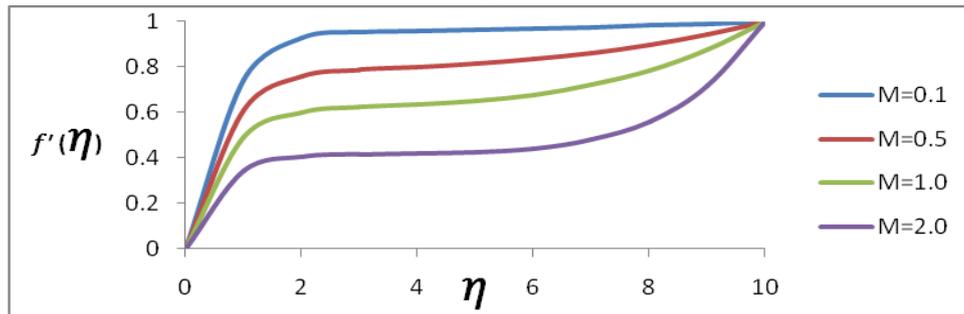


Figure 3: Effects of magnetic parameter on the velocity profile for Pr = 0.72, Sc = 0.24, Bi = 0.1 and K = 0.2

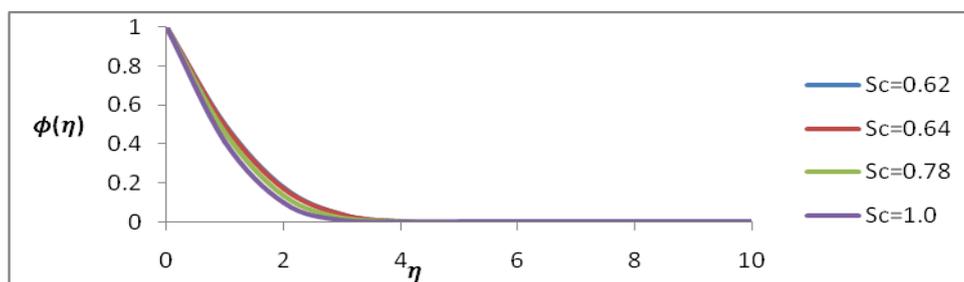


Figure 4: Effects of Schmidt number on the Concentration profile for Pr = 0.72, Bi = 0.1, M = 0.1 and K = 0.2

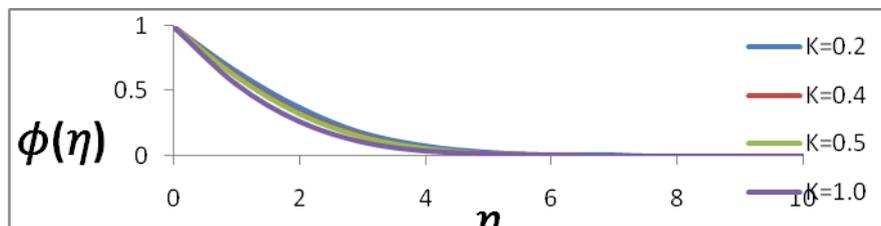


Figure 5: Effects of the reaction rate on the concentration profile for Pr = 0.72, Sc = 0.24, M = 0.1 and Bi = 0.1

Table1: Computation showing the variation in local skin friction, Nusselt number and Sherwood number

Pr	Sc	Bi	M	K	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.72	0.24	0.1	0.1	0.2	1.182584	0.086406	0.371633
2.71	0.24	0.1	0.1	0.2	1.182584	0.078958	0.371633
5.00	0.24	0.1	0.1	0.2	1.182584	0.074380	0.371633
7.10	0.24	0.1	0.1	0.2	1.182584	0.071414	0.371633
0.72	0.62	0.1	0.1	0.2	1.182584	0.086406	0.562154
0.72	0.64	0.1	0.1	0.2	1.182584	0.086406	0.569889
0.72	0.78	0.1	0.1	0.2	1.182584	0.086406	0.620426
0.72	0.24	0.5	0.1	0.2	1.182584	0.279857	0.371633
0.72	0.24	1.0	0.1	0.2	1.182584	0.388614	0.371633
0.72	0.24	10.0	0.1	0.2	1.182584	0.597639	0.371633
0.72	0.24	0.1	0.5	0.2	1.013737	0.085534	0.351810
0.72	0.24	0.1	1.0	0.2	0.859928	0.084470	0.331205
0.72	0.24	0.1	2.0	0.2	0.670990	0.082539	0.302268
0.72	0.24	0.1	0.1	0.4	1.182584	0.086406	0.423986
0.72	0.24	0.1	0.1	0.5	1.182584	0.086406	0.448447
0.72	0.24	0.1	0.1	1.0	1.182584	0.086406	0.557786

IV. CONCLUSIONS

The effects of chemical reaction on the stagnation point flow has been examined via the method of similarity transformation using Runge-Kutta iterative scheme together with a shooting technique. It is observed that the convective boundary conditions, the stagnation point, chemical reaction and the magnetic field have significant effects on the heat and mass transfer rate, velocity boundary layer thickness and the thermal boundary layer thickness.

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